MaGIC Workshop on Group Actions in Cryptography, Borgo Lanciano, 7th of May 2026

CORAL
Faster Isogeny Group Action For Post-Quantum NIKE

Joint with Andrea Basso, Giacomo Borin, and Sina Schaeffler

Ryan Rueger
IBM Research Zurich & Technical University of Munich

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Summary: $\Delta_\mathcal{O} \approx 2^{512}$ 🪦 $\Delta_\mathcal{O} \approx 2^{1024}$ 😟 $\Delta_\mathcal{O} \approx 2^{2048}$ 😊 $\Delta_\mathcal{O} \approx 2^{4096}$ 🤩 (Very Scientific!!1!)

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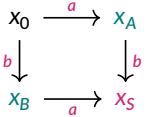

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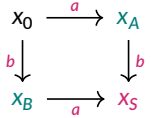

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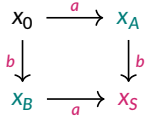

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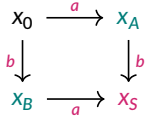

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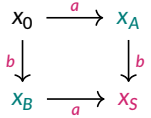

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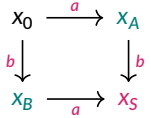

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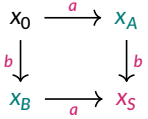

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Strategy Large Discriminant and "Small" Key-space (*e.g.* $\Delta_{\mathcal{O}} = 2^{4096}, |\mathcal{K}| = 2^{128}$)

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CSI-FiSH (Dimension 1)

Find equivalent ideal $\mathfrak{l}_1^{e_1} \cdots \mathfrak{l}_n^{e_n} \sim \mathfrak{c}$ of smooth norm

Requires computing the structure $\mathrm{Cl}(\mathcal{O}) \cong \mathbb{Z}^n/\Lambda$ (Superpolynomial: Limited to $\Delta_\mathcal{O} \approx 2^{500}$ 🪦)

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Problem

- Base prime characteristic for CSIDH must shape $p + 1 = \ell_1 \cdots \ell_n$
- Base prime characteristic for qt-PEGASIS must shape $p + 1 = c2^f$

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Yes their base prime is the same!

The most efficient instantiation of CORAL uses exactly the same parameters as qt-PEGASIS

Sampling $\mathfrak{a} \sim \mathfrak{b}$ such that $\mathrm{N}(\mathfrak{a}) + \mathrm{N}(\mathfrak{b}) = 2^e$

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which we can solve "efficiently" in practice (by isogeny standards)

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and how big does $N(a) + N(b) = 2^e$ have to be?

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1) $2^{e-1} + a$, $2^{e-1} - a$ behave like random integers

2) The events that $2^{e-1} + a$, $2^{e-1} - a$ are represented by a given class $[\mathfrak{c}]$ are independent

one can prove that the distribution of resulting classes $[(2^{e-1} - a, x + y\sqrt{-p})\mathcal{O}]$ is uniform

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Experimental data supports both $e \approx \log(p)/2 + \lambda + \varepsilon$ and uniformity

Comparison of selected isogeny-based group-action evaluations

			Time (ms)			
			$\approx \log(\Delta_{\mathcal{O}})$			
	Name	Lang.	512	1024	2048	4096
Unrestricted	KLaPoTi [PPS25]	Rust	2 360	17 500	138 000	
	PEGASIS [DEF+25]	Sage	1 900	5 290	21 300	121 000
	qt-PEGASIS [DEIV25]	Sage	1 200	3 250	11 800	58 000
	qt-PEGASIS [DD26]	C	73.6	419	2 940	22 000
Restricted	CSIDH [CLM+18]	C/asm	26.5	529		
	SQALE [CSCJR22]	C/asm	66.7	240	1 300	7 710
	dCTIDH [CHMR25]	C/asm			400	
	CSIDH-LDO [ZLL+26]	C/AVX			408	696
	OSIDH-LD [WLLZ26]	C/AVX				754
	CORAL	C	6.99	36.5	240	1 780
	CORAL	C/asm	4.43			

Benchmarks exclusively measure the isogeny computation; excluding norm equation solving, public key compression, or public key validation where applicable. Using variants `AVX-512IFMA-8w_r7` and `AVX-512IFMA-8w8w_r13` for CSIDH-LDO2000 and CSIDH-LDO4000 respectively. The dCTIDH variant used is $m = 6, \ell = 194$. The SQALE variant used is "MCR". `asm` denotes use of assembly-optimized arithmetic. AVX denotes use of platform-specific *Advanced Vector Extensions*. Data for CORAL and CSIDH collected over 1000 samples. Data for other algorithms collected over 100 samples. All benchmarks measured with AMD Ryzen 7 PRO 7840U@3.3GHz, boost and hyperthreading disabled. Times rounded to three significant figures.

Comparison of selected post-quantum NIKEs

Scheme	Lang.	Actively secure	|pk| (B)	KeyGen (ms)	SharedKey (ms)
Swoosh [GdKQ+24]	Rust Jasmin `asm`	No‡	221184	38.2	2.53
dCTIDH2047 [CHMR25]	C/`asm`	Yes	256	3840	479†
CSIDH-LDO2000 [ZLL+26]	C/AVX	No*	320§	439	427
CORAL2032-1155	C	Yes	256	594	240
CSIDH-LDO4000 [ZLL+26]	C/AVX	No*	576§	784	761
OSIDH-LD4000 [WLLZ26]	C/AVX	No*	576§	844	754
CORAL4092-2185	C	Yes	512	5480	1 780

Using variants `AVX-512IFMA-8w_r7` and `AVX-512IFMA-8w8w_r13` for CSIDH-LDO2000 and CSIDH-LDO4000 respectively. CSIDH-LDO variants include compression (KeyGen) and decompression (SharedKey). †With improved public key validation as described in [PRR+25, Tab. 4]. *Does not include public-key validation necessary to make the protocol actively secure. At time of writing, practically efficient public-key validation is still in progress [Hou25, Sec. 7]. §Using best possible theoretical approximation $2\log(p) + \log(\mathrm{Disc}(\mathcal{O}))$ bits for public-key sizes, not implemented sizes (here $r = 7$ (resp. 13) for 2048-bit (resp. 4096-bit) discriminant). ‡Whilst the full Swoosh protocol can be made actively secure though applying proofs-of-knowledge, the current (benchmarked) implementation does not include these; nor does the reported public-key size include the additional 89 KB space required for these proofs. `asm` denotes an implementation with assembly-optimized arithmetic. Data collected from 500 key exchanges (*i.e.* 1000 KeyGen and SharedKey calls). All benchmarks measured with AMD Ryzen 7 PRO 7840U@3.3GHz, boost and hyperthreading disabled. Times rounded to three significant figures.

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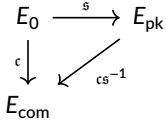
Example 1-bit identification scheme (core of CSI-FiSh Signature)



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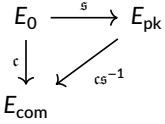

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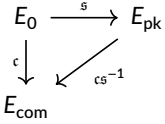

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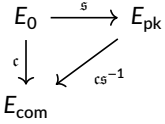

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Ongoing work

Can also compute 2-dim representation of $\mathfrak{c}\mathfrak{s}^{-1}$ to allow for 2-dimensional verification

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Applications

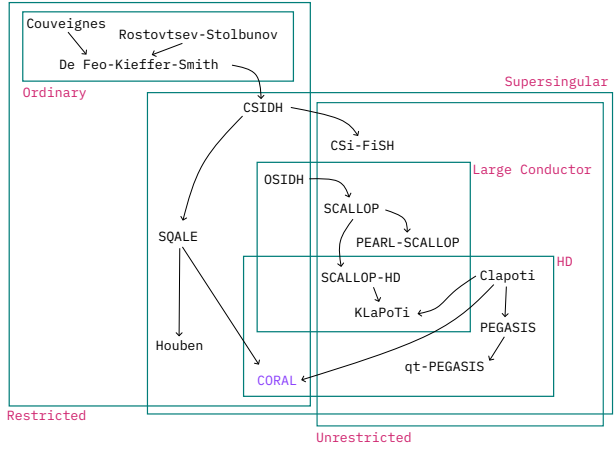
- Which protocols can we accelerate with CORAL? (*e.g.* for NIST's Threshold Call)

Thank you

CORAL to appear at Crypto'26

Paper will be on ePrint soon...

https://rueg.re/magic26



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